DEVELOPMENT OF DISCRETE CRACKS IN CONCRETE LOADED BY SHOCK WAVES

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Abstract *The nonlinear behaviour of concrete is affected by the development of cracks. The micro cracks are the initiation for the development of the macro cracks under different loadings. This work presents a discrete crack model with a cohesive crack zone to simulate the behaviour of concrete under a high dynamic load. In consideration of the strain rate effect and the Hugoniot-curve shock waves in concrete are calculated. The simulation of the blasted concrete results in a realistic crack pattern.*

Introduction

Simulation of high dynamic loading of concrete needs special numerical and material models. The development of shock waves and consequently the discontinuity in front of the shock wave have to be considered. Another challenge is the calculation of the concrete cracking.

The idea of this research is to use the discrete cracks with a cohesive crack model instead of a damage material model. The results of these calculations will be compared with experimental results of blasted concrete.

Element-free Galerkin Method

MLS Interpolation

Belytschko [2] proposed the element-free Galerkin method (EFG) which approximates a field by using a moving least-squares interpolation (MLS Interpolation). The following equation is used for the approximation of the displacement field

$$u^h(\mathbf{x}) = \sum_{i=1}^{n} \phi_i^k(\mathbf{x}) u_i = \mathbf{p}^T \cdot \mathbf{a} \tag{1}$$

The shape function ϕ is built from monomial functions $\mathbf{p}$. A linear 2-dimensional example for $\mathbf{p}$ is

$$\mathbf{p}(\mathbf{x}) = (1 \; x \; y)^T \tag{2}$$

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The vector **a** is calculated by the minimisation of the interpolation error J_i

$$J_i(\boldsymbol{a}, \boldsymbol{x}) = \sum_{i=1}^{n} w_i(\boldsymbol{x}) \cdot \left(u_i - p_i^T(\boldsymbol{x}) \cdot \boldsymbol{a}\right)^2 \tag{3}$$

By using the derivation of equation 3 the shape functions ϕ_i^k can be calculated. The weight function w_i depends on the distance $\| \mathbf{x} - \mathbf{x}_i \|$. The weight function can be written as

$$w_i(\mathbf{x}) = w_i(s) \text{ with } s = \frac{\| \mathbf{x} - \mathbf{x}_i \|}{h_i} \tag{4}$$

A common spline function can be used as the weight function

$$w(s) = 1 - 6s^2 + 8s^3 - 3s^4 \tag{5}$$

The size of the radius of influence h_i should be choosen so that 4 to 10 nodes are in a supported area.

The discretisation of the EFG-Method is similar to the finite element method by using the shape functions (equation 1).

Instead of a nodal integration a background integration is used. This results in an increased computing time but is necessary as the combination of a nodal integration and an explicit time integration shows problems with under integration.

MLS Interpolation and Cracks

Cracks can be implemented in EFG by cutting off the weight functions (and herewith the shape functions) at the location of the crack (Figure 1). The domain of the examined node is divided into two subdomains – subdomain B beyond the crack and subdomain A on the side of the node. In subdomain A the spline function w is the same as before, in subdomain B it is set to

$$w(\mathbf{x}) = 0 \text{ for all } \mathbf{x} \in B. \tag{6}$$

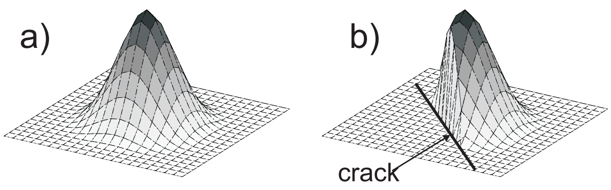

Figure 1: Weight function a) without crack b) with crack

Material Model

Two general material models for concrete can be distinguished: smeared and discrete crack models. In a smeared crack model the strains resulting from a crack are dispersed over one or more elements. In contrast to a discrete crack model the location of the crack is not stored. A discrete crack model helps to consider the fragmentation of the concrete for example after high dynamic loading. In the presented work discrete cracks are implemented with EFG. The use of discrete cracks with a fracture process zone makes it possible to use a material model without damage formulation.

There are two effects to be considered for the calculation of high dynamic loaded concrete: the building of shock waves and the strain rate effect.

Nonlinear stress-strain relation

Concrete responds to loading very nonlinear. In this work cohesive cracks are implemented to describe the cracking effects in the micro and meso scale. In a zone – called fracture process zone (FPZ) – the stresses between the crack sides decreases from tensile strength to zero. The length of the FPZ can be calculated with the crack energy. For the distribution of the stresses between the crack sides an exponential, a bilinear or a linear function can be used. The differences in numerical results using the various distributions of the stress are small. A bilinear distribution shows the best representation of the experiments.

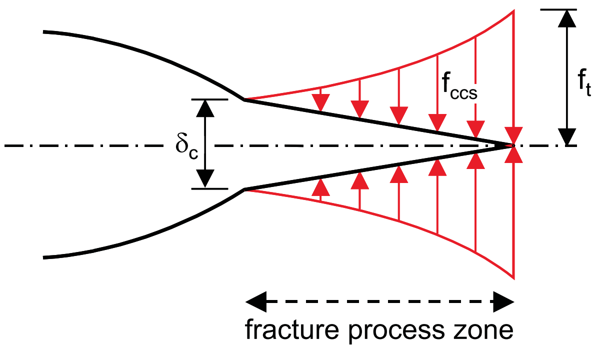

Figure 2: Fracture process zone

The following rules are used for the development of the cracks:

- A failure surface should be used for the decision if a crack has to be created or a crack is growing. For a tension failure a St. Venant criterion is applicable, but this failure surface doesn't show a good correlation by a two or three axial loading. Therefore the failure surface proposed by Hsieh, Ting and Chen [6] is used in this work.

- A fixed crack length is used for the development of the cracks.

- The direction of the crack growth is orthogonal to the direction of the principle stress.

To decide if a crack is developed the stresses at the crack tip are calculated by a MLS Interpolation. This is a non-local determination.

Hugoniot

The nonlinear volumetric stress-strain relation is the cause for the development of shock waves. The first part of the volumetric strain-pressure curve is the elastic part. The gradient in this part is the elastic compression modulus. By increasing volumetric strains the micropores in the concrete are damaged. The gradient of the stress-strain relation becomes smaller than the compression modulus – the waves moves more slowly. After the destruction of the micropores the stiffness of the concrete is getting higher due to the compaction of the material (Hugoniot). The increased stiffness is the reason for the development of the shock waves.

In the presented work a Y-function is used to consider the increase of stiffness (figure 3). The bulk modulus K has to be multiplied with this function. The increasing stiffness does not influence the shear modulus.

$$K_{tot} = Y(\epsilon_v) \cdot K \tag{7}$$

The shape of the Y-function is shown by Schmidt-Hurtienne [8]

$$Y = \begin{cases} \left[1 - a_v \cdot \left(1 - e^{-\frac{|\epsilon_v| - e_{v,th}}{e_v}}\right)\right] \cdot \left[1 + \left(\frac{|\epsilon_v| - e_{v,th}}{b_v \cdot e_v}\right)^2\right] & \text{for } \epsilon_v < -e_{v,th} \\ 1 & \text{for } \epsilon_v \geq -e_{v,th} \end{cases} \tag{8}$$

The following parameters are used

$$\begin{aligned} e_{v,th} &= 0.008 \\ e_v &= 0.0223 \\ a_v &= 0.7 \\ b_v &= 3.5 \end{aligned} \tag{9}$$

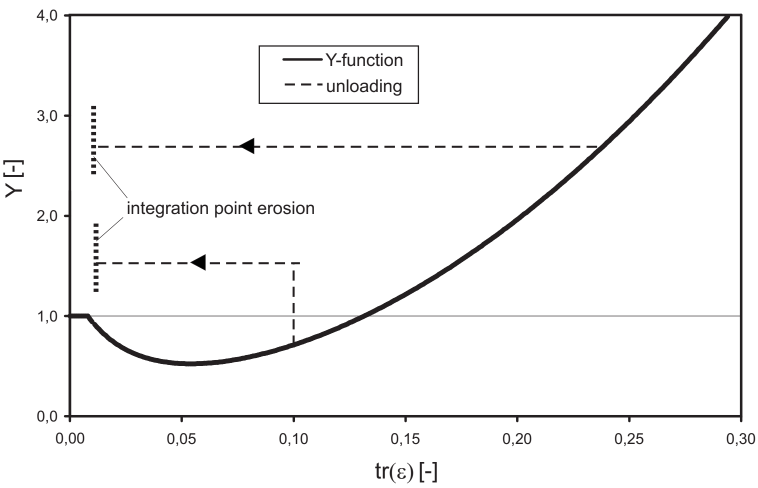

Figure 3: Hugoniot-curve – Y-function

The Y-function for unloading is set to

$$Y_{unloading} = \begin{cases} 1 & \text{for } \max Y_{loading} = 1 AND \min Y_{loading} = 1 \\ Y_{unloading,plast} & \text{for } \max Y_{loading} = 1 AND \min Y_{loading} < 1 \\ \max Y_{loading} & \text{for } \max Y_{loading} > 1 \end{cases} \quad (10)$$

The investigation shows that the numerical model represents the material by using $Y_{unloading,plast} = 1.5$. This unloading function is also shown in figure 3.

The damaging of the concrete under high hydrostatic loads has also to be considered. If the micropores of the concrete are destroyed, the concrete cannot bear to a tension load anymore. Furthermore, the shear modulus is reduced. If these effects are not considered, too less energy is dissipated in the system and the amplitude of the wave does not decrease.

The reduced shear modulus is implemented with a damage evolution following the proposal of Ruppert [7]

$$D_z = \left(\frac{\epsilon_v}{\epsilon_{v,max}} \right)^{\gamma} \quad (11)$$

with the hydrostatic strain ϵ_v and the hydrostatic strain capacity $\epsilon_{v,max}$ of the concrete. The parameter γ defines the shape of the damage. The investigations show that a ductile shape is suitable to represent the experimental data.

After the crushing of the micro pores the concrete cannot sustain a tension load anymore. The concrete reacts like a granular material. Therefore, if crushed concrete is loaded by tension, the integration points and the integration cells are deleted.

Strain rate effect

The tensile and the compression strength increase with increasing strain rates. This has been shown in multiple experiments for example by Bischoff [3]. If concrete is blasted the strain rate reaches values of $10^6\,\mathrm{sec}^{-1}$. It is not possible to get experimental results from concrete strength for strain rates higher than $100\,\mathrm{sec}^{-1}$. So the strength factor for strain rates beyond this point is hypothetical. In contrary to the CEB Bulletin [4] the strength factor should be limited to the maximal value resulting by the experiments.

Results

Shearing of a plate under detonation load

Plates loaded by heavy air blast waves fail in a shear failure instead of a bending failure. Experiments conducted by Albritton [1] with reinforced concrete show the crack pattern illustrated in figure 4.

A specimen (figure 4) is loaded by a triangular load-time function with a maximal load of $20\,\mathrm{MPa}$ and a rising time of $0{,}05\,\mathrm{msec}$. A background mesh with a distance between the nodes of $10\,\mathrm{mm}$ is used. The strain rate effect is considered.

Figure 5 shows the numerical crack pattern. The crack runs from the support into the specimen. Additional cracks develop on the right hand side of the support because of the missing reinforcement.

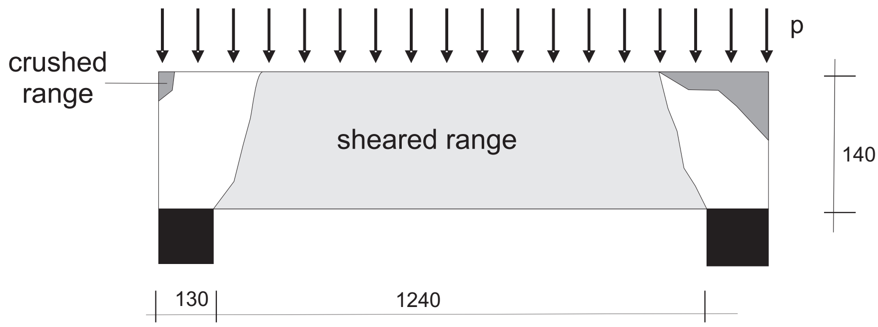

Figure 4: Experimental setup

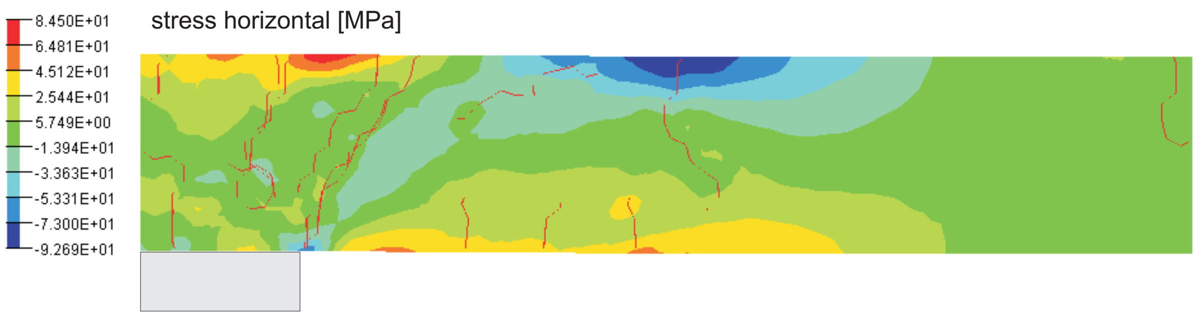

Figure 5: Crack pattern

Blasting of concrete

A contact detonation loads concrete with shock waves. At the Institute of Reinforced Concrete Structures and Building Materials at the University of Karlsruhe concrete slabs are loaded by an explosive to obtain the material parameters of high dynamic loaded concrete. The blasting results in a crater beneath the explosive. The concrete underneath the crater is highly compacted. Below this compacted range the concrete is damaged by cracks (Figure 7 – left). The aim of this work is to use the shown simulation model for the calculation of this crack pattern.

The simulations of the experiments use the following parts of the material constitution:

- EFG and discrete, cohesive cracks with an easy contact algorithm
- Limitation of the variation of the direction (necessary for EFG)
- Limitation of the crack velocity – Experiments from Curbach [5] show a limitation of the crack velocity to a value of 500 m/sec. Within the calculation the crack velocity is limited to this value.
- Y-function with unloading function
- Erosion of the integration cells
- Strain rate effect

The result shows that the amplitude of the wave decreases very fast. The decreasing of the pressure is shown in figure 6 for different values for the unloading parameter $Y_{unloading,plast}$. The experiments are well represented by using $Y_{unloading,plast} = 1.5$.

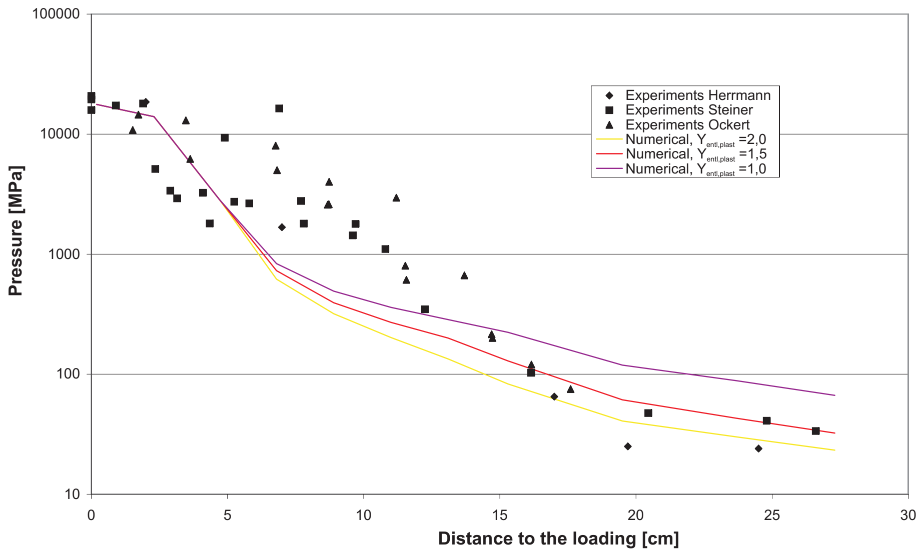

Figure 6: Development of the pressure by a contact detonation

The comparison of the cracks pattern in the concrete shows a good conformity between the experimental and the numerical results (Figure 7 – right).

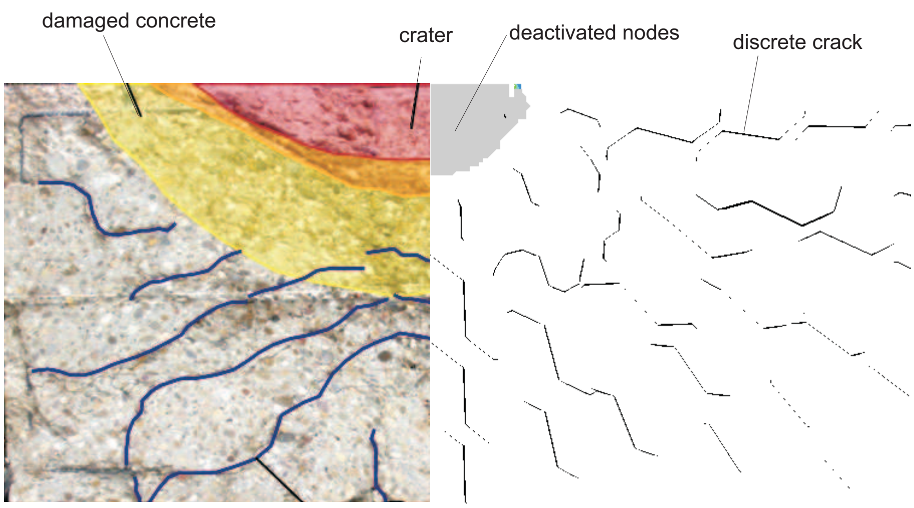

Figure 7: Cracks formation due to blast loading of concrete

Conclusions and Outlook

The element-free Galerkin method offers a possibility to model discrete cracks in concrete. The combination of the discrete cracks and a fracture process zone allows the description of the behaviour of concrete. The results of simulations of blasting show a good correlation with experimental data.

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